Problem 12-19) From Problem 15, part (b), we have $\boldsymbol{\nabla}\times(\psi\boldsymbol{A})=(\boldsymbol{\nabla}\psi)\times\boldsymbol{A}+\psi\boldsymbol{\nabla}\times\boldsymbol{A}$. Replacing $\boldsymbol{A}$, which, in general, is a function of position $\boldsymbol{r}$, with the constant vector $\boldsymbol{C}$, makes the curl term on the right-hand side of the above identity to vanish. We will then have $\boldsymbol{\nabla}\times(\psi\boldsymbol{C})=(\boldsymbol{\nabla}\psi)\times\boldsymbol{C}$. Applying the Stokes theorem to the left-hand-side of the above equation, we find

$$\int_{\text{surface}}\boldsymbol{\nabla}\times(\psi\boldsymbol{C})\mathrm{d}\boldsymbol{s}=\oint_{\text{boundary}}\psi(\boldsymbol{r})\boldsymbol{C}\cdot\mathrm{d}\boldsymbol{\ell}=\boldsymbol{C}\cdot\oint_{\text{boundary}}\psi(\boldsymbol{r})\mathrm{d}\boldsymbol{\ell}. \qquad (1)$$

On the right-hand-side, the surface integral may be somewhat simplified, as follows:

$$\int_{\text{surface}}[(\boldsymbol{\nabla}\psi)\times\boldsymbol{C}]\cdot\mathrm{d}\boldsymbol{s}=-\int_{\text{surface}}[(\boldsymbol{\nabla}\psi)\times\mathrm{d}\boldsymbol{s}]\cdot\boldsymbol{C}=-\boldsymbol{C}\cdot\int_{\text{surface}}\boldsymbol{\nabla}\psi(\boldsymbol{r})\times\mathrm{d}\boldsymbol{s}. \qquad (2)$$

The above expressions are valid for *any* (arbitrary) constant vector $\boldsymbol{C}$ and, moreover, they are equal to each other. We conclude that the coefficients of $\boldsymbol{C}$ on the right-hand-sides of Eqs. (1) and (2) must be the same, that is, $\int_{\text{surface}}\boldsymbol{\nabla}\psi(\boldsymbol{r})\times\mathrm{d}\boldsymbol{s}=-\oint_{\text{boundary}}\psi(\boldsymbol{r})\mathrm{d}\boldsymbol{\ell}$.